The Learning Process Methodology: A Universal Model of the Learning Process and Activity Design

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Abstract

The study of learning and instructional design has produced many overlapping models for the learning process and activity design. Specific to Process Education, the Learning Process Methodology (LPM) is inclusive of most elements of these models. The evolution of learning activity templates has produced components that reveal strong correspondence with this methodology. This paper will support each step of the LPM, show which components in activity design are justified from these steps, provide criteria for effective design of these components, and reveal the overlap of the LPM with other models for the learning process and learning activities. This will in turn show how universal this methodology is for both learning as a process and for the design of learning activities

Introduction

The Learning Process Methodology is an abstract generalization of the learning process. It provides one with the sequence of steps used by the strongest learners and can be applied to any learning activity. It has undergone slight changes since its inception. The contemporary version presented in Table 1 is from *Learning to Learn: Becoming a Self-Grower* (Apple, Morgan, & Hintze, 2013). For more information on the LPM see Apple, Ellis, and Hintze, 2016.

Instructional design occurs at different levels – program design, course design, activity design, and component de-

Table 1 The Learning Process Methodology

Step	Explanation
Stage 1: Preparing to Learn	
1 Why	Identify and explain your reasons for learning.
2 Orientation	Develop a systematic overview of what is to be learned.
3 Prerequisites	Identify necessary skills and background knowledge needed to perform the learning.
4 Learning Objectives	Set appropriate goals and objectives for the learning activity.
5 Performance Criteria	Determine specific desired outcomes used to measure and gauge performance.
6 Vocabulary	Identify and learn key terminology.
7 Information	Collect, read, and study appropriate resources.
Stage 2: Performing a Learning Activity	
8 Plan	Develop a plan of action to meet the performance criteria.
9 Models	Study and review examples that assist in meeting the learning objectives and performance criteria.
10 Thinking Critically	Pose and answer questions that stimulate thought and promote understanding.
11 Transfer/Application	Transfer knowledge to different contexts; apply knowledge in new situations.
12 Problem Solving	Use knowledge in problem solving situations.
Stage 3: Assessing and Building New Knowledge	
13 Self-Assessment	Assess use of the learning process and mastery of the material learned.
14 Research	Create and develop knowledge that is new and unique.

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sign. This paper focuses on learning activities and their components as the fundamental units of instructional design. Since an activity is intended to be a learning experience, the design of activities is inherently tied to the learning process and thus the LPM. Effective activity design templates, such as the POGIL design (POGIL, 2015), are based on the learning process, yet no explicit mention of the LPM has been made in their analysis (Hanson, 2007a, p. 281-284).

In judging the effectiveness of a learning activity, the intent of the individual components and their sequencing is paramount. So much variation exists in the labeling of these components that they must be mapped to the steps of the learning process, in this case the LPM, for proper analysis. We will consider the learning activity provided in Appendix A as a basis for components to be mapped to the LPM. As with any one example, certain niche components are missing but all categories of components are represented. The example activity happens to be for a Mathematics course, yet the analysis that follows should apply to all academic content areas.

A review of literature in the areas of both the learning process and activity design reveal a strong correspondence with the LPM. The models included for comparison are chosen for their familiarity in the field. Similar mappings can be made with many other models. A brief discussion of each model is provided below with a full description available in the cited source.

Notable Models for Activity Design

- **First Principles of Instruction**

  (Merrill, 2009)

  M. David Merrill developed the five first principles of instruction after reviewing various instructional design theories and looking for commonalities. The five principles can be summarized as:

  1. Problem-centered – Learners are solving real-world problems
  2. Activation – Relevant previous experience is activated
  3. Demonstration – The instruction demonstrates what is to be learned
  4. Application – Learners are required to use their new knowledge or skill to solve problems
  5. Integration – Learners are encouraged to integrate the new knowledge or skill into their everyday life

  The principles are used to show that there are relationships universal to all theories for instructional design. The model is used by educators and learning theorists to develop learning materials and has been supported with later research.

- **Kemp Instructional Design Model**

  (Morrison, Ross, & Kemp, 2001)

  Also known as the Morrison, Ross, and Kemp Model, this multidisciplinary theory is nonlinear and often uses a cyclic structure, though the components are numbered here to allow comparison to the LPM. The nine elements are:

  1. Identify instructional problems and goals
  2. Examine learner characteristics
  3. Identify subject content and analyze task components related to stated goals and purposes
  4. State instructional objectives for the learner
  5. Put content in a logical sequence
  6. Design strategies for learners to master the objectives
  7. Plan the instructional delivery
  8. Develop evaluation instruments to assess objectives
  9. Select resources to support instruction and learning activities

  The model is not prescribed as a methodology and encourages users to work in multiple areas or omit areas if appropriate. The nine components also work within a larger structure of planning, support, management, evaluation, and revision that Morrison has since incorporated into a more current version of the theory.

- **POGIL Classroom Activity Elements**

  (POGIL, 2015)

  Process Oriented Guided Inquiry Based Learning uses collaborative learning to increase understanding. The POGIL organization provides research and resources that promote the theory and practices used. They outline nine elements in a typical activity as:

  1. Title – what the students will learn
  2. Why? – puts the activity in context
  3. Prerequisites – prior knowledge or resources needed for the activity
  4. Learning Objectives – information used to determine whether success has been achieved
  5. The Model – work used to show the learner proper technique

6. Guided Inquiry Questions – questions that guide the student to understanding
7. Exercises – straightforward applications of the important concepts developed within the activity
8. Problems – a situation where the student will not know immediately what to do
9. Closure – self-assessment and reflection

Similar to the LPM, the POGIL guidelines correspond to specific components in a learning activity. The language used is also similar to that of Process Education as David Hanson, one of the founders of the POGIL movement, also contributed to the *Faculty Guidebook*.

- **The Systems Approach Model**

(Dick, Carey, & Carey, 2004)

This classic model is also known as the Dick and Carey Model and was originally published in 1978. It consists of the ten components:

1. Identify Instructional Goals
2. Conduct Instructional Analysis
3. Analyze Learners and Contexts
4. Write Performance Objectives
5. Develop Assessment Instruments
6. Develop Instructional Strategy
7. Develop and Select Instructional Materials
8. Design and Conduct Formative Evaluation of Instruction
9. Revise Instruction
10. Design and Conduct Summative Evaluation

As the name implies, Dick and Carey viewed the learning process as a system where many parts were interconnected. The model is also nonlinear and often uses a directed graph of flowchart to express the relationships between the components.

Notable Models of the Learning Process

- **ARCS Model of Motivational Design Process**

(Keller, 2010)

John Keller developed the ARCS model which consists of the four components:

1. Attention – Get the attention of the learner by arousing perceptions and inquiry
2. Relevance – Make the learning relevant, goal-oriented, and familiar
3. Confidence – State learning requirements, give significant opportunity, and make learners personally responsible.
4. Satisfaction – Maintain equity through the use of intrinsic and extrinsic feedback

The model applies motivational theory to the learning process. The first two components are usually emphasized as more important than the latter two.

- **Gagne's Nine Conditions of Learning**

(Gagne, 1970)

Robert Gagne's model is also known as the nine events of instruction and includes:

1. Gaining Attention
2. Informing Learners of Objectives
3. Stimulating Recall of Prior Learning
4. Presenting the Stimulus
5. Providing Learning Guidance
6. Eliciting Performance
7. Providing Feedback
8. Assessing Performance
9. Enhancing retention and transfer

The steps are ordered and used in a linear fashion. Gagne's model blurs the line between learning theory and instructional design. While based on the psychology of learning, the model is often used by instructional designers to create activities.

- **Kolb's Experiential Learning Cycle**

(Kolb, 1984)

David Kolb used previous learning theories to create this 4 part cycle of learning. It consists of:

1. Having a concrete experience
2. Reflective observation of the experience
3. Abstract conceptualization – concluding from the experience and reflection
4. Active experimentation – trying out what you have learned

While there is an order to the steps the fourth step leads back to the first and one can technically start at any point in the cycle.

Analysis of the LPM

With the LPM and seven comparable models now introduced, the LPM is ready to be analyzed. First, Table 2 summarizes the correspondence between steps of the LPM and components in a learning activity.

For examples of these components types, see the sample activity in Appendix A. While activity component names will vary, their mapping to the steps of the LPM based on their purpose should be similar.

In Table 3 we see the correspondence between the steps of the LPM and the components of the seven models outlined earlier. If there was significant similarity in the step of the LPM and the component of the compared model, then it was included in Table 3. This integration is useful for the step by step analysis of the LPM that follows.

Notice that there is no other theory that includes all steps of the LPM and yet every method has some overlap. Often the steps will have the same name or be synonymous with a component in another method. It is also worth mentioning that Steps 1, 11, and 13 are included in all seven models and Step 6 is not given an explicit component in any of them.

For each step of the Learning Process Methodology, the following analysis is offered (after Table 3):

- The step is justified by support from outside sources
- The step is correlated to the components of a learning activity, criteria are provided for designing effective components
- The step is mapped to steps in the aforementioned models

Table 2 Mapping of LPM to Components of Learning Activity

Stage of the LPM	Steps of the LPM	Learning Activity Components
Stage 1 ***Preparing to Learn***	Step 1 Why	Title, Why, Purpose
	Step 2 Orientation	Concept Map, Table of Contents, Discovery
	Step 3 Prerequisites	Prerequisites, What Do You Already Know?
	Step 4 Learning Objectives	Learning Objectives, Learning Goals, Learning Outcomes
	Step 5 Performance Criteria	Performance and Criteria (Success Criteria)
	Step 6 Vocabulary	Vocabulary, Terminology, Notation, Developing Language in [content area], Learning the Language of [content area]
Stage 2 ***Performing a Learning Activity***	Step 7 Information	Information, Resources, Methodologies, Common Errors, Models
	Step 8 Plan	Plan, Tasks, Facilitation Plan
	Step 9 Models	Models, Examples, Your Turn
	Step 10 Thinking Critically	Critical Thinking Questions, Exploratory Questions, Key Questions
	Step 11 Transfer/Application	Skill Exercises, Demonstrate Your Understanding, Hardest Problem
	Step 12 Problem Solving	Making it Matter, Problem Solving, Problems
Stage 3 ***Assessing and Building New Knowledge***	Step 13 Self-Assessment	Learning to Learn Mathematics (or other knowledge area), Self-Assessment, Identify and Correct Errors, Troubleshooting, Reflection on Learning
	Step 14 Research	Divergent Critical Thinking Question, Undergraduate Research

Table 3 Mapping of LPM Steps to Other Models of Instructional Design

LPM Step	Merrill	Kemp	POGIL	Dick & Carey	ARCS	Gagne	Kolb
1	Problem Centered	Examine learner characteristics	Why?	Analyze learners and contexts	Attention and Relevance	Gain Attention	Concrete experience
2	Activation	Identify subject content and analyze task components related to stated goals and purposes		Instructional analysis			Concrete experience
3	Activation		Prerequisites	Analyze learners and contexts	Relevance and Confidence	Stimulate recall of prior learning	Reflective observation
4		State Instructional objectives	Learning Objectives	Identify Instructional Goals	Confidence	Inform learner of objectives	
5				Write Performance Objectives			
6							
7	Demonstration	Select resources to support instruction		Develop and Select Instructional materials		Present stimulus material	Concrete experience
8		Plan the Instructional Delivery		Develop Instructional strategy		Provide learner guidance	
9	Demonstration		The Model			Provide learner guidance	Reflective observation
10			Guided Inquiry Questions			Eliciting Performance	
11	Application	Develop evaluation instruments to assess objectives	Exercises	Develop assessment instruments	Confidence and Satisfaction	Enhancing Retention and transfer	Abstract conceptualization
12	Problem Centered and Integration		Problems		Satisfaction	Enhancing Retention and transfer	Active experimentation
13	Integration	Develop evaluation instruments to assess objectives	Closure	Design and Conduct Formative Evaluation	Satisfaction	Assessing Performance	Reflective observation
14							Active experimentation

Step 1: Why

The first step of the LPM identifies and explains the reasons for learning. The idea that adults need to know why before they fully engage in the learning process is well supported (Knowles, 2005). A quality learning activity begins with a title followed by the purpose statement, both of which support the first step of the methodology (Table 2). The title serves as a label for the knowledge item(s) focused on in the experience and introduces the topic to the learner in relevant language, thus motivating the need for the activity in a personal sense. An effective title is succinct and memorable, setting the tone for the discourse that follows. A purpose statement highlights what is to be learned along with its various contextual facets. Aspects of effective purpose statements include: (1) Clarity achieved through a non-technical approach to the topic, (2) motivation through the defining of a relationship to the "big picture" in the content area, and (3) relevancy in the learner's personal and professional life. When the learner recognizes the activity will improve their life, the process is already geared toward a more significant level of learning (Rogers, 1969). A strong purpose statement will get the learner's attention and make the learning relevant, coinciding with the attention and relevance steps in the ARCS Model of Motivational Design and the first of Gagne's Nine Events of Instruction – gaining attention (see Table 3).

Step 2: Orientation

A systematic overview is developed to identify what is to be learned and how it relates to other knowledge. It has been shown by Hicks and Klimoski (1987) that a proper preview of the learning activity results in a greater commitment to and satisfaction with the subsequent learning. A discovery activity supports the Orientation step of the LPM as it sets the context and area of knowledge prior to dispensing information (see Table 2). Dewey (1944) argued long ago that new ideas should be introduced with "something to do, not something to learn", comparing each learning activity to an act of experimentation performed by a scientist in a laboratory. To best encourage learners to explore the essential content's core, an interesting, intriguing, and fun activity is advised for inclusion into the learning experience. An interactive discovery activity agrees well with the first step in Kolb's Experiential Learning Cycle – concrete experience (See Table 3). Thus by exploring and experimenting a successful learning environment is established (Bransford, Brown, & Pellegrino, 2000).

Step 3: Prerequisites

Prerequisites identify the transferable knowledge needed to perform strongly in the learning experience. The paradigm of learning as a "successive transition between knowledge states" is well known from classic cognitive research (Dochy, 1995). The hierarchical nature of knowledge within a content area is well supported, e.g., in Mathematics the learning of higher-level rules is dependent on the mastery of lower-level rules (Gagne, 1970). By incorporating a component that asks the question "What do you already know?" a learning activity takes advantage of knowledge from prior courses or life experiences and relates it to the new learning (see Appendix A). These questions that draw on previous knowledge and the discovery activity prepare the learner for the reading and models that follow. This component goes with Merrill's second principle of instruction – Activation, and Gagne's third event of instruction – stimulate the recall of prior learning. Ultimately, the richest resource for adult learners may be the relating and interpreting of personal experiences (Lindeman, 1926).

Step 4: Learning Objectives

The ideal learning process is prefaced with statements of intent. Stakeholders in the process must know ahead of time what they will learn so that they can validate this later on (Dick et al., 2004). John Dewey, Ralph Tyler, William Torrey Harris, and others have previously established the importance of learning objectives (Block, 2012). Most modern models also align in some way (See Table 3). A set of no more than 3 orthogonal learning objectives, meaning that overlap is minimized, is recommended. Other best practices have been suggested for writing learning outcomes in 2.4.5 of the *Faculty Guidebook* (Beyerlein, Davis, & Apple, 2007). Obviously, there should be clear alignment between the learning objectives for the activity and the learning objectives for the course and program. However, since the measurement of learning is done via "the performance exhibited by the learner" (Gagne, 1970) proper assessment and evaluation of the activity requires that criteria be specified for that performance.

Step 5: Performance and Criteria

While the learning objectives list what is intended to be learned in an activity, the performance criteria provide the metric for both faculty and students to make assessments and evaluations of the outcomes later on. Performance criteria are acknowledged as a requirement for high quality assessment (Stiggins,

2001). In many models of instructional design, the support of this step overlaps with step 4, though some emphasize the theory of performance (Table 3). In particular, Gagne (1970) exhorts the deposition of the performance criteria to the learner as the first step in a learning activity. To help improve the quality of the criteria, one should suspend for the time being the measurement process that will follow. A flexible and holistic approach to this crucial step is detailed in 4.1.8 of the *Faculty Guidebook* (Baehr, 2007)

Step 6: Vocabulary

The identification and learning of vocabulary phase also includes the study of terminology and notation. According to Davis (1944), the study of vocabulary is essential to reading comprehension, the foundation of reading for learning. Knowing the specific vocabulary terms and their precise meaning allows learners to interpret resources and communicate effectively in speech and in writing. While it is common for textbooks to include important vocabulary as part of a summary or index, the salient inclusion of these items in the early stages of the learning activity is warranted (Richardson & Morgan, 1990). The activity component's name is often dependent upon the content area (e.g., Learning the Language of Mathematics in the example provided in Appendix A); however, its necessity permeates across the curriculum. A strategy for studying relevant terminology should be included in the course or program design as it has been seen that "students will skip over unfamiliar words" otherwise (McMurray, Laffey, & Morgan, 1979). A typical best practice is to have learners use the words in context or write definitions in their own words (i.e., create their own glossary).

Step 7: Information

The last step in the preparing to learn stage of the learning process is to collect, read, and study appropriate background information. In some ways this is always in a learning activity so it precludes gratuitous justification. It is generally agreed upon in academia that reading should be a "lifetime experience" (Richardson & Morgan, 1990). While the medium of information and resources varies now more than ever, it still serves as the locus of instructional design. Reading assignments from print resources now compete with digital text and video recordings and transmissions as the primary repository for learner information. Regardless of medium, information for a learning activity should always be made easily available, relevant, and variegated to foster maximum learning (Gagne, 1970). Methodologies, generalizations of process knowledge, are essential to effective learning in some content areas. By analyzing the generalized steps and applying them to similar examples, internalization of the methodology is facilitated (Leise & Beyerlein, 2007). Organizational tools (e.g., concept maps) are better suited for conceptual knowledge, highlighting the importance of classifying the knowledge for prior to the designation of the information in the learning activity (Quarless, 1970). Gagne specifically emphasizes organization as a key criterion when describing resources for effective learning activities. A component in the learning activity that emphasizes common errors with examples, which is technically misinformation, aligns well with this step of the LPM. Students are bound to make mistakes and most of them are clustered around a few common errors. In order for students to learn validation of their work, they should see the analysis of incorrect thinking and misconceptions.

Step 8: Plan

In the same way steps 1-7 prepare the learner for the learning activity, steps 8-12 guide the participant while they perform the learning activity. The first step in the performance is to develop a plan to meet the performance criteria. With a performance in mind and criteria developed in step 5, the learner should follow the proverbial advice that tells us to *stop and think before you act*. To ensure the best possible performance, a well thought out plan must be developed first (Black, Harrison, Lee, Marshall, & Wiliam, 2004). We see this acknowledged in stage six of the Dick and Carey Model and the seventh key element in Kemp's model. In the loosest sense, the plan is a set of directions for how to complete the activity. With active learning or collaborative learning, this includes items such as: defining teams and team roles, sharing of pre-activity learning, analyzing models, in-class discovery exercises, answering critical thinking questions, and assessing the performance. See the *Faculty Guidebook* 3.2.5 for more detailed information on development of facilitation plans. The fact is students "need to know how the learning will be conducted" in order to be prepared and the plan should be written with clarity and detail for the performance that follows (Knowles, 2005). If learners have a copy of this plan or are even groomed to take part in its development, then they will take more responsibility for their learning.

Step 9: Models

Providing examples of work that demonstrates the learning objectives and performance criteria is the ninth step in the learning process. One of the keys to becoming a master learner is to realize that the

methods of experts are more important to master than the knowledge itself (Dewey, 1944). Effective learners use and apply models to understand all forms of knowledge (Knowles, 2005). Comparison to other experts in education reveals comparable ideas, e.g., Merrill's Demonstration and Gagne's Provide Learner Guidance. While traditional lectures technically fulfill this role, active learning suggests having learners read from prepared models when possible to build self-learning habits. This better supports a collaborative learning facilitation plan as well as accounting for differences in styles and speed of comprehension. At this point the learner has some familiarity with the basic methodology, so more involved examples can be used as models; they serve both as objects of inquiry for the critical thinking questions, which are often difficult due to their abstraction of ideas, and a resource for the problem solving that will follow. Models should always be paired with analysis or validation from a subject matter expert, often after initial dissection by the learners alone.

Step 10: Thinking Critically

To depose and respond to critical questions is at the heart of the universal learning process. The questions will promote basic understanding and elevate the learning to Level 2 – Comprehension on Bloom's taxonomy. The literature abounds with support for critical thinking in the learning process. It is considered a significant step to understand highly complex issues (Brookfield, 1986). It is even the sixth basic principle in Constructivism to support and challenge the thinking of the learner (Savery & Duffy, 1996). Dewey (1944) goes even further to assert that only by wrestling with these questions and figuring out the answers do we actually *think*. The questions are essential to guide the student's inquiry through the models to produce understanding and meaning, to address misconceptions that might form, to serve as interplay between steps in the methodology, and to deal with subject specific reasoning issues. This makes the experience active and encourages them to eventually ask their own questions when learning. By juxtaposing questions with the models or information already developed in steps nine and seven, the retention of knowledge is significantly improved. Best practices for critical thinking questions suggest six to ten questions that are relevant, growth-oriented, and logical (Hanson, 2007b). The first two to three questions should be directed and focus on Level 1 knowledge. The next three to six questions should be convergent and build Level 2 knowledge. The final question should be divergent and thus open-ended. This last question opens up to research level learning projects as well as providing a balance for advanced teams that would normally finish an assignment early.

Step 11: Transfer/Application

After thinking critically, the learner should test their understanding by transferring it to varying contexts and applying it in new situations. The underlying motivation of all adult learners is the awareness of situations that can serve as applications of the knowledge (Knowles, 2005). This "testing ideas against alternative contexts" is the seventh constructivist principle (Savery & Duffy, 1996) and the key to elevating knowledge from Level 2 to Level 3 (Nygren, 2007). While many books recognize this step with problems simply called Applications, the sample in the appendix uses the label *Demonstrate Your Understanding* (See Appendix A). In this component the structure is again crucial and should be scaled in difficulty with scrutiny. As students deal with more challenging problems they should start with a familiar context, then move into less familiar and finally unfamiliar contexts in at least three problems of increasing difficulty. This grows the confidence of the learners, thus supporting the ARCS theory. In content areas where problem solving is integral, the quintessential component for the transfer/application is the eponymous *Hardest Problem* (See Appendix A), where learners come up with the hardest problem they can think of and then try to solve it or explain why it cannot be solved. In doing so, the learner identifies the key parameters and how their variation affects the solution, ultimately learning to generalize the problem. This technique of abstract conceptualization is also the third stage in Kolb's experiential learning cycle.

Step 12: Problem Solving

While Step 11 has learners applying knowledge into new contexts, the problem solving step pits the learner against complex problems like those worked on by experts in the field. Problem solving combines knowledge to elevate the level of learning and generalize an entire set of situations as a single type (Gagne, 1970). Though challenging, the relevance to career credentials has been shown to improve motivation (Clark, Dobbins, & Ladd, 1993). Problem solving is the eighth type of learning according to Gagne, the fourth step in the experiential learning cycle according to Kolb, the fourth Constructivist principle, the fifth step in guaranteed learning, Merrill's fifth principle of instruction, and provides satisfaction to the learner according to the ARCS model. In the

component called *Making it Matter* (Appendix A), students are given a real-life problem that deals with the topic at hand. In general education courses, where learners are all pursuing mutually disjoint career paths, the focus can be academic or generated from the duties of an individual in a human community. The motivation created by the appearance of utility often supersedes any actual value, as techniques used in any industry are mercurial.

Step 13: Self-Assessment

Having finished the learning performance, the mastery of the learning objectives and fulfilling of the performance criteria can be assessed. Self-Assessment has the learner give themselves feedback on the performance with a growth mindset, i.e., the intention of improving future performances. Constructivism supports this type of reflection on content and the learning process, as do other models (Table 3). Learning how to learn is a skill necessary for all workers to stay competitive (Knowles, 2005). A self-assessment on the learning process has the learner identify strengths in their performance, provide plans for improving in future performances, and share insights gained from the experience. This should be done in the form of a positive attitude that avoids harsh self-judgement. A separate assessment of the content-specific mastery (e.g., *Learning to Learn Mathematics* in our example in Appendix A) has the students reflect on the processes used in that subject, making the learning experience a transformative one.

Step 14: Research

The final step in the learning process opens the individual to the creation of new knowledge from what has been learned. As students become master learners their pursuit of knowledge will transition from guided discovery to independent exploration fueled by self-interest (Dewey, 1944). While most activities do not formally allocate components to this step it is not absent from education entirely. Long-term research projects assigned for outside of class can potentially fulfill this step, especially if the student is the catalyst for the assignment and decides on the topic. Research learning can also occur in class when students seriously approach the divergent critical thinking question. Ultimately the step is needed for a complete model of the learning process but does not require a corresponding component in many educational activities.

Concluding Remarks

The Learning Process Methodology was developed for the purpose of increasing the engagement of educators with learners (Apple et al., 2016), but can also be seen as a tool for designing learning activities. Specifically, the value and arrangement of the components in a learning activity can be validated by their agreement with the LPM. Since the LPM has been extended as a tool for self-directed learning, facilitating, and assessing, a similar analysis could be done for these contexts. As evinced by the POGIL movement, the creation of activity design templates with a strong emphasis on the learning process not only leads to effective education for learners but also sustains communities of professional development (POGIL, 2016). Despite the apparent universal nature of the LPM, it should be continuously analyzed, assessed, and compared with new models as the research on learning sciences evolves so that the next iteration of the LPM can lead to stronger educational outcomes.

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2.3 Evaluating a Formula

DISCOVER THINK APPLY SOLVE REFLECT

Purpose

The role this topic plays in quantitative reasoning

The most common quantitative exercise is evaluating a result based upon a formula where values are substituted for the formula's variables. In fact, mathematical models are fundamental in many disciplines and these models are most often a formula or a series of formulas. Once you understand how to evaluate and manipulate formulas, you will not only have the confidence to take full advantage of the useful formulas available in the world around you, but also be able to trust that your resulting calculations are correct.

Learning Goals

What you should learn while completing this activity

1. Correctly set up a mathematical model or use the correct formula.
2. Evaluate formulas (algebraic expressions) given values for some or all of the variables.
3. Document and explain the role of units in evaluating formulas and validating the results.

Discovery

Finding out for yourself

In exploring the world around you, consider that many common activities could be represented by formulas: How much do you spend at the grocery store weekly? How much did you get paid this month? How much does each member of a group have to contribute for paying the dinner check? How did you balance and validate that your checkbook is correct? How do you calculate your current GPA? Can you write and use a formula to make each of these calculations? As stated in the title of this experience, what does it really mean to *evaluate a formula*?

What Do You Already Know?

Tapping into your existing knowledge

1. What is a formula?
2. What formulas have you used in the past?
3. If a formula has three variables, and you want to "evaluate the formula," for how many of the variables do you need to know values?
4. What is the Order of Operations?
5. What is a code word or phrase (mnemonic) you can use to help yourself remember the Order of Operations?

Mathematical Language

Terms and notation

formula — defines a quantity that can be calculated by providing specific value(s) for all the variables in the defined expression

unit analysis — when evaluating a formula, you can check your work by carrying the units of each number through the formula. If your evaluation is correct, then some units should divide out, leaving you with the correct units for the answer. This form of validation is called *unit analysis* or *dimensional analysis*.

METHODOLOGY — USING UNIT ANALYSIS WHEN EVALUATING A FORMULA

Scenario: Compute the perimeter of a rectangle with a length of 3 feet and a width of 5 feet.

Step	Explanation	Watch it Work!
1. Identify the formula you need to use	*Use the right formula in its most useful form.*	$P = 2(L + W)$
2. Identify the units in the problem	*Make a list of all relevant units mentioned in the problem. Identify which units measure the same things.*	feet
3. Substitute given values for the variables	*Include units as you make the substitutions in the formula.*	$P = 2(3 \text{ feet} + 5 \text{ feet})$
4. Simplify units	*Divide out any common units that appear in both the numerator and denominator of multiplied fractions.*	no units to simplify
5. Convert to same units	*Multiply by a fraction equal to 1 to make units that measure the same thing match.*	units already match
6. Simplify units	*Divide out any common units that appear in both the numerator and denominator of multiplied fractions.*	no fractions
7. Perform all operations	*Keep the remaining units as you carry out the operations.*	$P = 2(8 \text{ feet}) = 16 \text{ feet}$
8. Validate your answer and its units	*Work backwards from the answer to the original values.*	$16 \text{ feet} = 2(L + W)$ $16 \text{ feet} = 2(3 \text{ feet} + W)$ $16 \text{ feet} = 6 \text{ feet} + 2W$ $10 \text{ feet} = 2W$ $W = 5 \text{ feet}$ ✓

Scenario: Find the distance traveled if a car is driving 50 miles per hour for 90 minutes.

Step	Watch it Work!
1. Identify the formula you need to use	$d = rt$
2. Identify the units in the problem	*miles, hours, minutes; hours and minutes both measure time*

Step	Watch it Work!
3. Substitute given values for the variables	$d = r \bullet t$ $d = \frac{50 \text{ miles}}{1 \text{ hour}} \bullet 90 \text{ minutes}$
4. Simplify units	*no units to simplify*
5. Convert to same units	$d = \frac{50 \text{ miles}}{1 \text{ hour}} \bullet 90 \text{ min} \bullet \frac{1 \text{ hour}}{60 \text{ min}}$
6. Simplify units	$d = \frac{50 \text{ miles}}{1 \cancel{\text{hour}}} \bullet 90 \cancel{\text{min}} \bullet \frac{1 \cancel{\text{hour}}}{60 \cancel{\text{min}}}$
7. Perform all operations	$d = 75 \text{ miles}$
8. Validate your answer and its units	$d = 75 \text{ miles} = rt$ $75 \text{ miles} = r(90 \text{ minutes})$ $r = \frac{75 \text{ miles}}{(90 \text{ minutes})} = \frac{75}{90} \bullet \frac{\text{miles}}{\text{minutes}}$ $r = \frac{5}{6} \bullet \frac{\text{miles}}{\text{minutes}} = \frac{5}{6} \bullet \frac{\text{miles}}{\text{minutes}} \bullet \frac{60 \text{ minutes}}{\text{hour}}$ $r = \frac{5}{6} \bullet \frac{\text{miles}}{\cancel{\text{minutes}}} \bullet \frac{60 \cancel{\text{minutes}}}{\text{hour}} = \frac{5 \bullet 60}{6} \bullet \frac{\text{miles}}{\text{hour}} = 50 \frac{\text{miles}}{\text{hour}}$ ✓

YOUR TURN!

Scenario: Find the speed of a car in kilometers per hour, if it is driving at 1 mile per minute

Oops! Avoiding Common Errors

- **Not calculating/validating both the value and the unit**

Example: What is the area of a rectangle that is 4 feet wide and 9 feet long?

A = width × length *A* = 4 × 9 = **36** incorrect (units left out)	*A = width × length* *A* = 4 ft × 9 ft = **36 ft** incorrect (units not calculated)	*A = width × length* *A* = 4 ft × 9 ft = **36 ft²** correct (feet × feet = square feet)

Why? Answers that have units must contain both the number and the unit. This means that both must be calculated and validated. If the correct answer to a problem is "30 miles per hour," an answer of "30" is as incorrect an answer as "30 kilometers per hour" is.

- **Justifying an answer instead of validating it**

Example: How much does the gas cost, in dollars, to drive 500 miles in a car that gets 23.4 miles to the gallon, if gas costs \$3.67?

T = total cost of gas

P = price for 1 gallon of gas

M = miles traveled

$$mpg = \frac{\text{miles}}{\text{gallon}}$$

Step 1: Identify formula: $T = M(mpg)P$

Step 2: (units were identified previously)

Step 3: Substitute units: $T = 500(23.4)(3.67)$

. . . .

Step 8: Validate units: $T = \text{mile}\left(\frac{\text{gallons}}{\text{mile}}\right)\left(\frac{\$}{\text{gallon}}\right)$

Why? Often we justify the answer we want instead of validating whether it works or not. In this case, the student set the problem up incorrectly (in Step 1), but instead of using the correct units (in Step 8) to validate the work, the student flipped mpg so that the previous mistake was justified. Had the student actually validated the units, it would have become obvious that there was an error in the formula used, as the correct units do **not** validate the formula. When you validate, you're testing to see if an answer is correct, not trying to make it correct.

Are You Ready?

Before continuing, you should be able to ...

I can...	**OR** *Here's my question...*
☐ correctly multiply a fraction by an integer	
☐ substitute correctly in a formula	
☐ apply the Order of Operations correctly	
☐ use unit analysis correctly in a formula	

DISCOVER **THINK** APPLY SOLVE REFLECT

Plan

How to complete the activity

1. Compare the answers to Your Turn in practicing the use of the methodology.
2. Answer the Critical Thinking Questions.
3. Complete the remainder of this activity (from Demonstrate Your Understanding through Assessing Your Performance) on your own, or as directed by your instructor.

Model(s)

Exemplars and representations

Using a Formula

The formula for distance in terms of rate and time is: $d = r \bullet t$.

We can find the value of d if $r = 30\ \dfrac{\text{miles}}{\text{hour}}$ and $t = 2$ hours.

$$d = r \bullet t$$

$$d = \left(30\ \frac{\text{miles}}{\text{hour}}\right)(2\ \text{hours})$$

$$d = 60\left(\frac{\text{miles}}{\text{hour}} \bullet \text{hours}\right) = 60\left(\frac{\text{miles}}{\cancel{\text{hour}}} \bullet \frac{\cancel{\text{hours}}}{1}\right)$$

$$d = 60\ \text{miles}$$

Critical Thinking Questions

Developing your understanding

1. In what order do you perform the steps when you use the Order of Operations?
2. Which step in the Order of Operations do you find the most difficult to carry out? Why?
3. What is the purpose of the Order of Operations?
4. How is the fraction bar like parentheses?
5. When evaluating an expression and substituting in specific values for variables, how do you ensure that the values are substituted in correctly?
6. How does a formula differ from an expression?
7. Why are the units required when evaluating a formula?

DISCOVER THINK **APPLY** SOLVE REFLECT

A Successful Performance

Successful application of your learning looks like this

As you begin to apply what you've learned, you should have a good idea of what success looks like.

A SUCCESSFUL PERFORMANCE

I set up and evaluate formulas. I...

- Choose and set up the correct formula
- Accurately substitute the known values
- Correctly compute both numbers and units

Demonstrate Your Understanding

Apply it and show you know in context!

1. Evaluate the following expressions for the given value(s).

   a) $3x^2 - 5x + 4$ for $x = -2$ b) $5(r^2 - 3s) - r + 3$ for $r = -2$ and $s = 4$

2. The formula for the volume of a cylinder in terms of its radius and height is given by the formula $V = \pi r^2 h$. Find the volume of a cylindrical oil tank with a radius of 7 meters and a height of 20 meters.

3. The formula for the intensity of light, I, in terms of the distance, d, from the source of the light, is given by the equation $I = \dfrac{k}{d^2}$.

   Find the intensity of light, I, when $d = 10$ meters and the constant $k = 350$ lumens • meters^2.

Hardest Problem

*How hard **can** it be? Can you still use what you've learned?*

Based on the Model, the Methodology, and the Demonstrate Your Understanding (DYU) problems in this activity, create the ***hardest*** problem you can. Start with the hardest DYU problem in this experience and by contrasting and comparing it with the other DYU problems, play "What if" with the different conditions and parameters in the various problems.

Can you still solve the problem? If so, solve it. If not, explain why not. What is it that makes a problem where you evaluate a formula a difficult problem to solve?

What are the conditions and parameters that make a problem involving evaluating a formula a difficult problem to solve?

Troubleshooting

Find the error and correct it!

Saroya was helping a local artist with a new installation. Her job was to fill a series of glass cylinders with colored water. She had managed all the smaller cylinders but was now faced with a cylinder taller than she was. Because she was dying the water herself to the artist's exact specifications of "1.5 ounces of dye per gallon of water," it was critical that she know how much water each cylinder would contain. She got out a tape measure, found a step ladder, and determined that the last cylinder was 1.5 feet in diameter and 8 feet tall. At right are her calculations for the volume of the cylinder.

$$V = \pi r^2 h \quad \text{and} \quad r = \frac{d}{2} \quad \text{so} \quad V = \pi\left(\frac{d}{2}\right)^2 h$$

$$V = \pi\left(\frac{1.5 \text{ feet}}{2}\right)^2 \bullet (8 \text{ feet}) \bullet \frac{12 \text{ inches}}{1 \text{ foot}}$$

$$V = \pi(0.75 \text{ feet})^2 \bullet (8 \text{ feet}) \bullet \frac{12 \text{ inches}}{1 \text{ foot}}$$

$$V = \pi(0.75 \cancel{\text{feet}})^2 \bullet (8 \cancel{\text{feet}}) \bullet \frac{12 \text{ inches}}{1 \cancel{\text{foot}}}$$

$$V = \pi(0.5625 \text{ in}^2) \bullet (96 \text{ inches})$$

$$V = 3.14 \bullet (0.5625 \text{ in}^2) \bullet (96 \text{ inches})$$

$$V = 169.56 \text{ in}^3$$

Once she had the volume, she checked her smart phone and found that 1 gallon = 231 cubic inches. Suddenly, she knew she'd done something wrong. There was no way THIS cylinder held less than 1 gallon of water.

DISCOVER THINK APPLY **SOLVE** REFLECT

Making it Matter

Solving problems in your life

- The loyalty program has been pervasive: Thrifty rental car gives a free rental day for every 16 rental days purchased; Food Warehouse gives a 1% rebate on non-alcohol purchases, Market Foods is giving cash rebates on all purchases other than liquor and cigarettes. What is the formula for the refund or savings you get each month for a program that you or a person you know is involved in?
- What is the formula for the cost of gas you buy at your local gas station? How much will 5.7 gallons cost?
- What is the formula for how much you tip at a local restaurant?
- What is the formula for postage on a letter mailed at a United States Post Office? How much does it cost to send a 5 oz. letter by First Class mail?

Learning to Learn Mathematics

Reflecting on and appreciating your learning

1. Do you think about a formula and why it works the way it does once you've used it a few times?
2. How do you know that what you calculated through using a formula is really correct?
3. What can you do about using a formula when not all the variables have known values?

Assessing Your Performance

Assessing your performance as a learner

Review the description of **A Successful Performance** and assess your own performance against the standard offered there.

- How and why was your performance **SUCCESSFUL**?
- How can you **IMPROVE** your performance, making it more successful? What *concrete steps* do you need to take to make each improvement?